

Violation of Cauchy-Schwarz inequalities by spontaneous Hawking radiation in resonant boson structures

J. R. M. de Nova, F. Sols and I. Zapata

Departamento de Física de Materiales, Universidad Complutense de Madrid, E-28040 Madrid, Spain

(Dated: January 29, 2013)

The violation of a classical Cauchy-Schwarz (CS) inequality is identified as an unequivocal signature of spontaneous Hawking radiation in sonic black holes. This violation can be particularly large near the peaks in the radiation spectrum emitted from a resonant boson structure forming a sonic horizon. As a function of the frequency-dependent Hawking radiation intensity, we analyze the degree of CS violation and the maximum violation temperature for a double barrier structure separating two regions of subsonic and supersonic condensate flow. We also consider the case where the resonant sonic horizon is produced by a space-dependent contact interaction. In some cases, CS violation can be observed by direct atom counting in a time-of-flight experiment. We show that near the conventional zero-frequency peak, the decisive CS violation cannot occur.

PACS numbers: 03.75.Kk, 04.62.+v, 04.70.Dy, 42.50.-p

The emission of Hawking radiation from the horizon of a black hole is an intriguing prediction of modern physics [1]. Due to the extremely low effective temperature, its detection in a cosmological context is unlikely to be achieved in the foreseeable future. However, it was noted by Unruh [2, 3] that Hawking radiation (HR) is an essentially kinematic effect that could be observed on a laboratory scale at temperatures which, while still too low, lie within conceivable reach. For a quantum fluid passing through a sonic horizon (i.e., a subsonic-supersonic interface), it has been predicted [4–10] that, even at zero temperature, phonons will be emitted from the horizon into the subsonic region. Attempts have been made to observe HR in an accelerated Bose-Einstein (BE) condensate [11]. An alternative route may be provided by a quasi-stationary horizon, which can be achieved by allowing a confined large condensate to leak in such a way that the outgoing beam is dilute and fast enough to be supersonic [12, 13].

Hawking radiation is a fundamentally quantum phenomenon that results from the impossibility of identifying the vacuum of incoming quasiparticles with that of outgoing quasiparticles [14]. Specifically, the incoming vacuum is a squeezed state of outgoing quasiparticles. In this respect, it has been long recognized in quantum optical contexts [15, 16] that correlation functions characterizing the electromagnetic radiation satisfy Cauchy-Schwarz (CS) type inequalities that can however be violated in the deep quantum regime. Thus, violation of CS inequalities is generally regarded as a conclusive signature of quantum behavior. By contrast, detection schemes based on the space correlation function [6] show no signal difference between spontaneous and thermal (stimulated) Hawking radiation processes [17].

An additional advantage of the focus on the violation of CS inequalities is that it permits to distinguish the specific squeezed character of HR from the general properties of coherent collective behavior. Measurements of

space correlation functions [6], or phonon [18] or atom [12] intensity spectra, would not allow for such a distinction. The reason is that the same Bogoliubov-de Gennes equations describe two different phenomena: linearized collective motion and quantum quasi-particle excitation. Collective motion is imprinted on the coherent (condensed) part of the wave function, which does not describe the squeezed zero-point dynamics of Bogoliubov quasi-particles. Importantly, we propose that spontaneous HR can be tested by a CS violation involving two specific outgoing channels, one traveling against the flow in the subsonic region and the other one dragged by the flow on the supersonic side. CS violation due to other phenomena will appear in other correlation functions.

It has recently been noted that HR could be observed more easily in contexts where the predicted spectrum is not thermal but peaked around a discrete set of frequencies [9, 12]. This could be the case in a sonic horizon formed by a double-barrier structure [12] with an effective potential $V(x) = Z[\delta(x) + \delta(x - d)]$ lying between the subsonic and supersonic asymptotic regions. Optical analogs can show similar peaked structures [19]. The purpose of this paper is to show that, despite the remaining difficulties, the violation of CS inequalities in Hawking radiation is comparatively much easier to observe near the peaks characteristic of resonant radiation. The quest for CS violation here proposed complements those approaches relying on the direct detection of entanglement, as applied to inflationary cosmology [20], black holes [21], general relativistic quantum fields [22], or black-hole (BH) analogs [23, 24].

We focus on the properties of the normalized second-order correlation function, defined for light as [15, 16]

$$g_{ij}^{(2)}(\tau) \equiv \frac{\langle \hat{a}_i^\dagger(0) \hat{a}_j^\dagger(\tau) \hat{a}_j(\tau) \hat{a}_i(0) \rangle}{\langle \hat{a}_i^\dagger(0) \hat{a}_i(0) \rangle \langle \hat{a}_j^\dagger(0) \hat{a}_j(0) \rangle}, \quad (1)$$

where $\hat{a}_i(t)$ is the Heisenberg operator for photon mode i , and the average is quantum-statistical. The correlation

function for classical light is obtained by removing the quantum average and replacing the Heisenberg operators $\hat{a}_i(t)$ by complex numbers. The following inequalities can be proven for classical light:

$$1 \leq g_{ii}^{(2)}(0), \quad (2)$$

$$g_{ii}^{(2)}(\tau) \leq g_{ii}^{(2)}(0), \quad (3)$$

$$\left[g_{ij}^{(2)}(\tau) \right]^2 \leq g_{ii}^{(2)}(0) g_{jj}^{(2)}(0), \quad (i \neq j). \quad (4)$$

These inequalities are satisfied not only by any classical state but also by quantum thermal states at high temperature. They are also satisfied by chaotic and coherent states.

The violation of any of the above inequalities is a signature of deep quantum behavior. States violating (2) are said to show sub-Poissonian statistics. Violation of (3) reflects anti-bunching. Expression (4) is a CS inequality. States which violate it at $\tau = 0$ are said to exhibit two-mode sub-Poissonian statistics. In general, the proof of (4) requires the system to be described by a positive (Glauber-Sudarshan) P function. Some quantum states such as two-mode squeezed states may not satisfy this condition and thus can violate (4). That could be the case in a collision between two BE condensates [25].

References 8, 12, 13, 17 have addressed the existence and possible detection of HR in bosonic condensates. Figure 1 shows the dispersion relation of the scattering channels following the mode notation of Refs. 12, 13, 17, to which the reader is referred for further details. A particular type of Hawking radiation often considered in analog systems is the emission of u -out phonons as originated in the anomalous transmission from the $d2$ -in channel. Hereafter, the operator $\hat{\gamma}_{i,\alpha}$ destroys a quasiparticle in mode i - α , with $i = u, d1, d2$ and $\alpha = \text{in, out}$. The dependence of $\hat{\gamma}_{i\alpha}(\omega)$ on the quasiparticle frequency ω will often be understood. As is conventional in finite temperature HR setups, we assume averages are taken for a thermal distribution of incoming quasiparticles, so that $\langle \hat{\gamma}_{i,\text{in}}^\dagger(\omega) \hat{\gamma}_{j,\text{in}}(\omega') \rangle = n_i(\omega) \delta_{ij} \delta(\omega - \omega')$, where

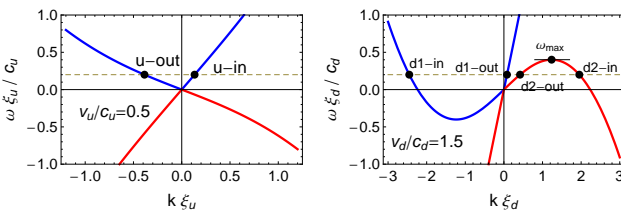


FIG. 1: Dispersion relation on the subsonic (left, upstream) and supersonic (right, downstream) sides. The blue/red branches correspond to positive/negative normalization. As in Refs. 12, 13, 17, d/u denotes downstream/upstream. Here, $\xi_{u/d}$ denotes the asymptotic healing length, $c_{u/d}$ and $v_{u/d}$ the sound and flow velocities, and ω_{max} the frequency above which no Hawking radiation can be generated.

$n_i(\omega) = [\exp(\hbar\Omega_i(\omega)/k_B T) - 1]^{-1}$ and $\Omega_i(\omega)$ is the comoving frequency corresponding to the mode i -in at the laboratory frequency ω .

We consider the equal-time second-order correlation function for the outgoing quasiparticle operators

$$\Gamma_{ij} \equiv \langle \hat{\gamma}_{i,\text{out}}^\dagger \hat{\gamma}_{j,\text{out}}^\dagger \hat{\gamma}_{j,\text{out}} \hat{\gamma}_{i,\text{out}} \rangle > 0, \quad (5)$$

and define $\theta_{ij} \equiv \Gamma_{ij}/\sqrt{\Gamma_{ii}\Gamma_{jj}}$, $\Theta_{ij} \equiv \Gamma_{ij} - \sqrt{\Gamma_{ii}\Gamma_{jj}}$, noting that the CS inequality (4) is violated if and only if

$$\theta_{ij} > 1 \text{ (or } \Theta_{ij} > 0). \quad (6)$$

Thus, we may use θ_{ij} (Θ_{ij}) as a relative (absolute) figure of merit to quantify the degree of CS violation.

We define the complex vector $\alpha_i^\dagger \equiv (\sqrt{n_u} S_{iu}, \sqrt{n_{d1}} S_{id1}, \sqrt{n_{d2} + 1} S_{id2})$, where S_{ij} is the element ij of the scattering matrix S characterizing the transition from j -in to i -out [26] and which obeying the pseudo-unitary condition $S^\dagger \eta S = \eta \equiv \text{diag}(1, 1, -1)$. Wick's theorem allows us to write

$$\begin{aligned} \Gamma_{uu} &= 2|\alpha_u|^4 \\ \Gamma_{d2d2} &= 2(|\alpha_{d2}|^2 - 1)^2 \\ \Gamma_{ud2} &= |\alpha_u^\dagger \cdot \alpha_{d2}|^2 + |\alpha_u|^2(|\alpha_{d2}|^2 - 1). \end{aligned} \quad (7)$$

Making use of (7), the CS inequality (4) for outgoing quasiparticles can be rewritten as

$$|\alpha_u^\dagger \cdot \alpha_{d2}|^2 \leq |\alpha_u|^2 (|\alpha_{d2}|^2 - 1), \quad (8)$$

which, due to the negative term within the bracket, can be violated some times. Interestingly, the very possibility of violating the CS inequality (4) is a direct consequence of the anomalous character of the scattering process $d2$ -in $\rightarrow u$ -out, because the $u/d2$ channel has positive/negative normalization. In fact, for the (normal) conversion $d1 \leftrightarrow u$, we obtain that (4) amounts to $|\alpha_u^\dagger \cdot \alpha_{d1}|^2 \leq |\alpha_u|^2 |\alpha_{d1}|^2$, which is always satisfied.

The inequality (8) and pseudo-unitarity lead, after a lengthy calculation, to the equivalent relation

$$\begin{aligned} |S_{ud2}|^2 (1 + n_u + n_{d1} + n_{d2}) &\leq \\ |S_{d1u}|^2 n_{d1} n_{d2} + |S_{d1d1}|^2 n_u n_{d2} + |S_{d1d2}|^2 n_u n_{d1} \\ + |S_{d2d1}|^2 n_u + |S_{d2u}|^2 n_{d1}. \end{aligned} \quad (9)$$

A similar inequality can be derived for the other anomalous process, $d1 \leftrightarrow d2$ (Andreev reflection [27]), by interchanging u and $d1$ in (9).

In the conventional ($\omega = 0$) peak of the Hawking spectrum $|S_{ud2}(\omega)|^2$, the scattering matrix elements diverge as $S_{ij}(\omega) \sim 1/\sqrt{\omega}$ with i arbitrary and $j = d1, d2$. By contrast, $S_{iu}(\omega)$ in the same limit ($\omega \rightarrow 0$) tends to a nonzero constant. On the other hand, the only occupation factor which diverges is $n_u(\omega) \sim 1/\omega$, because $\Omega_u(\omega)$ is the only comoving frequency that vanishes for small ω . From pseudo-unitarity it follows $|S_{ud2}|^2 - |S_{d2d1}|^2 =$

$|S_{d2u}|^2 - |S_{d1d2}|^2$. We conclude that (9) cannot be violated in the $\omega \rightarrow 0$ region. This argument relies only on the presence of a constant flow connecting the subsonic and supersonic asymptotic regions, and not on other details of the scattering structure.

The inequality (9) is however manifestly violated at temperature $T = 0$ and $\omega \neq 0$ provided $S_{ud2} \neq 0$, which further reflects the direct link between CS violation and Hawking radiation. Complementarily, the condition (6) is equivalent to

$$\theta_{ud2} = \frac{|S_{d2d2}|^2 - 1/2}{|S_{d2d2}|^2 - 1} > 1, \quad \Theta_{ud2} = |S_{ud2}|^2 > 0, \quad (10)$$

where pseudo-unitarity has again been invoked. The condition (10) is guaranteed to be satisfied because pseudo-unitarity requires $|S_{d2d2}| > 1$ whenever $S_{ud2} \neq 0$. Finally, we note that $\theta_{d1d2} = \theta_{ud2}$ and $\Theta_{d1d2} = |S_{d1d2}|^2$ at zero temperature.

A device producing HR works like a non-degenerate parametric amplifier, which is known to generate squeezing from vacuum. However, sources other than vacuum may also generate squeezing and ultimately CS violation [16]. In particular, the absolute amount of CS violation for a given frequency often increases initially as the temperature is raised from zero, eventually reaching a maximum and decreasing to zero at high temperatures, as can be guessed from Eq. (9). Therefore, one may wonder to what extent the CS violation here contemplated would provide conclusive evidence of spontaneous (zero-point) HR radiation. The answer is yes, as we argue below.

Wick's theorem can again be invoked to write Γ_{ij} in terms of first-order correlation functions such as $\langle \hat{\gamma}_{i,\text{out}}^\dagger \hat{\gamma}_{j,\text{out}} \rangle$ or $\langle \hat{\gamma}_{i,\text{out}} \hat{\gamma}_{j,\text{out}} \rangle$, here generically referred to as ρ_{ij} . The thermal contribution is $\rho_{ij}^{\text{th}} \equiv \rho_{ij} - \rho_{ij}^0$, with ρ_{ij}^0 the zero-temperature value (average over incoming vacuum). If one neglects the zero-point contributions by approximating $\rho_{ij} \simeq \rho_{ij}^{\text{th}}$, then one arrives at a modified version of (9) where only the terms quadratic in the n_i 's survive. The resulting inequality is always satisfied. We conclude that CS violation requires vacuum fluctuations.

We may define T_v as the highest temperature at which (9) is violated [(6) is satisfied] and try to identify some trends in its behavior. We note that, if the upstream current is small (kinetic contribution to total chemical potential on subsonic side is small), then in the central regions of the Figs. 2-3 ($\omega \sim \omega_{\text{max}}/2$) the comoving frequencies of the various channels are comparable (call them Ω') and all smaller than (typically a fraction of) the comoving chemical potential on the subsonic side, $\Omega' \lesssim \mu$. The resonant BH configurations, first discussed in [12], have the important advantage of displaying HR peaks at nonzero frequencies which generally lie in the interesting region near $\omega_{\text{max}}/2$.

If the conversion from negative to positive normalization is weak, then $|S_{d2d2}|$ is close to unity and from

pseudo-unitarity it can be proven that all the S -matrix elements appearing in (9) are small except for the relation $|S_{d1u}|^2 + |S_{d1d1}|^2 \simeq 1$. If $L \equiv -\log |S_{ud2}| \gg 1$ we obtain a low violation temperature, $T_v \sim \Omega'/L \ll \mu$.

Conversely, we find that for the highest HR peaks (where $|S_{ud2}| \gg 1$), then either $|S_{ud2}| \simeq |S_{d2u}| \gg |S_{d2d1}|$ or $|S_{ud2}| \simeq |S_{d2d1}| \gg |S_{d2u}|$. At the same time, $|S_{ud2}| \gg |S_{d1j}|$ for all j , which implies that the violation temperature $T_v \sim \Omega' |S_{ud2}|^2 / S_{d1}^2 \gg \mu$, where $S_{d1} \equiv \max_j \{|S_{d1j}|\}$. Thus, $T_v(\omega)$ is expected to be large near the peak frequency ω_0 . Non-resonant structures can also show CS violation at nonzero temperatures. However, even if the relative violation (as measured by θ_{ud2}) is significant, the absolute amount of violation (as measured by Θ_{ud2}) is negligible, as discussed later.

Another important advantage of resonant peaks is that, at their relatively high frequency, the phononic signal is approximately proportional to the atomic signal, which can be directly measured in a time-of-flight (TOF) experiment. Assuming that one can separate the subsonic and supersonic TOF signals, near the peak at $\omega = \omega_0$, and neglecting finite-size effects, the atom operators can be approximated as:

$$\hat{c}_u(p_u \equiv q_u + k_{u,\text{out}}) \sim \hat{\gamma}_{u,\text{out}}(\omega) \quad (11)$$

$$\hat{c}_d(p_{d1} \equiv q_d + k_{d1,\text{out}}) \sim \hat{\gamma}_{d1,\text{out}}(\omega) \quad (12)$$

$$\hat{c}_d(p_{d2} \equiv q_d - k_{d2,\text{out}}) \sim \hat{\gamma}_{d2,\text{out}}(\omega), \quad (13)$$

where $\hat{c}_{u/d}(k)$ annihilates an atom of momentum k on side u/d . Here, $k_{i,\text{out}}$ is the comoving quasiparticle momentum at the laboratory frequency ω in the i -out channel, and $q_{u/d}$ is the condensate momentum per atom on each side. The proportionality factors not shown in (11-13) cancel in the CS violation condition (6) for θ_{ud2} and θ_{d1d2} .

For the approximation (12-13) to apply, it is necessary that, for scattering processes with output on the supersonic side, the CS-violating S -matrix elements must dominate over the S -matrix elements at different frequencies but with the same laboratory momentum p_i , implicitly defined in (11-13). On the subsonic side, the approximation (11) requires the peak to be large, $|S_{ud2}(\omega_0)|^2 \gg 1$, because it has to stand out above the background of the depletion cloud [12].

From (10) and the pseudo-unitarity relation $|S_{ud2}|^2 + |S_{d1d2}|^2 + 1 = |S_{d2d2}|^2$, it may appear that a shortcoming of a large peak is its small relative degree of CS-violation (as measured by θ_{ud2}). However, this does not imply that the experimental signal is necessarily small. Quite the opposite, the absolute amount of violation (as measured by Θ_{ud2}) can be quite large, as (10) directly reveals. A similar analysis can be performed for the other anomalous process, $d1 \leftrightarrow d2$.

Whether or not the conditions for the approximation (11-13) are satisfied, we can define, in analogy to (5), the

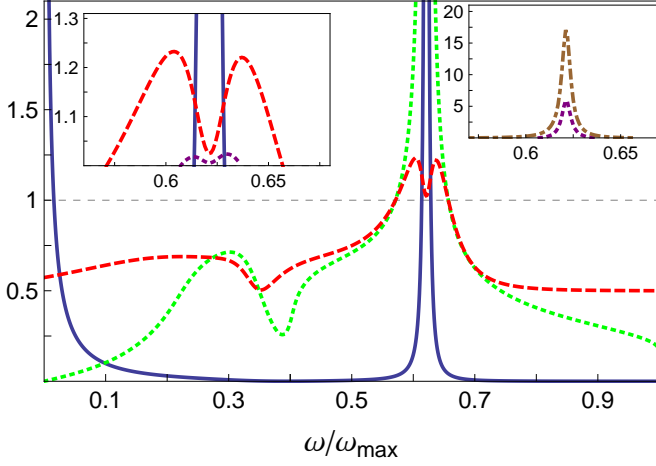


FIG. 2: Hawking radiation for a condensate leaking through a double barrier structure like that studied in Ref. 12. The strength of both delta barriers is $Z = 2.2\hbar^2/m\xi_u$. They are separated by a distance $d = 3.62\xi_u$. The flow is such that $q_u\xi_u = 0.01$ and $\omega_{\max} = 0.99\mu/\hbar$. Solid blue: zero-temperature Hawking radiation spectrum $|S_{ud2}(\omega)|^2$. Dashed red: $\theta_{ud2}(\omega)$ at temperature $T = \mu/k_B$, where μ is the comoving chemical potential on the subsonic side. Dotted green: maximum violation temperature $T_v(\omega)$ in units of μ/k_B . Not shown in the figure, $T_v(\omega)$ rises up to $T_v(\omega_0) \simeq 21\mu/k_B$, where $|S_{ud2}(\omega_0)|^2 \simeq 8$. Left inset: Zoom of the peak region, with $T_v(\omega)$ removed and $z_{ud2}(\omega)$ (relative atom CS violation) added (dotted purple). Right inset: same as left inset; it shows $\Theta_{ud2}(\omega)$ (dashed-dotted brown) and $Z_{ud2}(\omega)$ (dotted purple), which measure the absolute amount of CS violation in the phonon and atom signals.

atomic correlation functions:

$$G_{ud2}(\omega) \equiv \langle \hat{c}_u^\dagger(p_u) \hat{c}_d^\dagger(p_{d2}) \hat{c}_d(p_{d2}) \hat{c}_u(p_u) \rangle, \quad (14)$$

and similarly for the other G_{ij} , where $i, j = u, d1, d2$, and $p_i(\omega)$ is defined in (11-13). It can be shown that a sufficient condition for (6) is:

$$z_{ij} > 1, \quad Z_{ij} > 0, \quad (15)$$

where $z_{ij} \equiv G_{ij}/\sqrt{G_{ii}G_{jj}}$ and $Z_{ij} \equiv G_{ij} - \sqrt{G_{ii}G_{jj}}$.

In Fig. 2 we plot, for a double delta-barrier structure, the zero temperature HR spectrum $|S_{ud2}(\omega)|^2$, together with $\theta_{ud2}(\omega)$ (at $k_B T = \mu$) and the violation temperature $T_v(\omega)$. The left inset magnifies the peak region and includes $z_{ud2}(\omega)$, which measures the relative amount of CS violation in the atomic signal. The right inset shows $\Theta_{ud2}(\omega)$ and $Z_{ud2}(\omega)$, i.e., the absolute amount of CS violation by the phonon and atom signals. The latter can be directly measured in a TOF experiment. These graphs indicate that the considered structure is a promising scenario for the unambiguous detection of spontaneous Hawking radiation.

The inset of Fig. 3 shows the corresponding curves for a single delta-barrier black-hole configuration. The HR spectrum is unstructured, with a single peak at $\omega=0$.

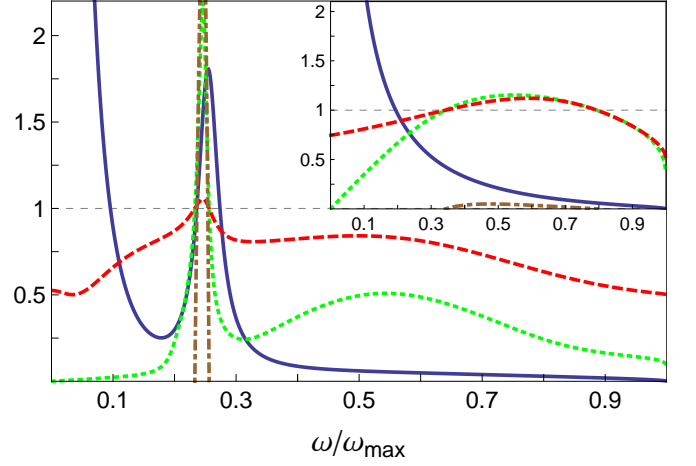


FIG. 3: Same as Fig. 2 for a structure without barriers but with two sharp variations in the local speed of sound, which takes the successive values $c_u, 0.5c_u, 0.2c_u$. The intermediate region has a length $12\xi_u$. The flow is such that $q_u\xi_u = 0.7$ and $\omega_{\max} = 0.21\mu/\hbar$. This is a resonant generalization of the model first studied in Ref. 6, where only one sound-speed discontinuity was considered. Inset: Same curves for a setup with a single delta barrier of strength $Z = 0.62\hbar^2/m\xi_u$ and a uniform interaction, with flow $q_u\xi_u = 0.3$ and $\omega_{\max} = 0.6\mu/\hbar$. Atomic CS violation is not found for these structures.

The CS inequality can be violated at the relatively high temperature $T = \mu/k_B$ within a considerable frequency range. However, due to the smallness of $|S_{ud2}|$, the atomic signal for a given momentum includes contributions from different phonon frequencies and from the depletion cloud, which implies that the approximation (11-13) cannot be used. Moreover, the smallness of the absolute phonon violation signal, together with the absence of atomic CS violation, suggests that non-resonant structures are not good candidates for the observation of CS violation.

The main Fig. 3 shows the same curves as in Fig. 2 but for a different type of resonant structure where spatial dependence of the interatomic contact interaction is tuned to produce step-like variations in the local speed of sound. This model is inspired in the setup proposed in Ref. 6, where the subsonic-supersonic interface is induced by a single step in the profile of the speed of sound. The absolute CS violation is considerably smaller here than in Fig. 2, but could still be observable. The sufficient atom inequality (15) is not satisfied.

In conclusion, we find that CS violation in the strongly peaked Hawking radiation emitted by a double-barrier sonic black-hole may provide a convenient route to the unambiguous observation of the zero-point contribution. In some setups, the violation of CS inequalities is large enough to be detectable by the direct observation of second-order correlation functions in an atom time-of-flight of experiment. These advantages are absent in non-resonant structures such as that formed by a single-

barrier sonic black-hole. In particular, the relevant CS violation cannot occur near the conventional, zero-frequency Hawking radiation peak universally shown by all one-dimensional BH structures.

We thank D. Guery-Odelin, R. Parentani and C. Westbrook for valuable discussions. Support from MICINN (Spain) through grant FIS2010-21372 and from Comunidad de Madrid through grant MICROSERES-CM (S2009/TIC-1476) is also acknowledged.

-
- [1] S. W. Hawking, *Nature* **248**, 30 (1974); *Commun. Math. Phys.* **43**, 199 (1975).
 - [2] W. G. Unruh, *Phys. Rev. D* **14**, 870 (1976).
 - [3] W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981).
 - [4] U. Leonhardt, T. Kiss, and P. Öhberg, *J. Opt. B* **5**, S42 (2003).
 - [5] U. Leonhardt, T. Kiss, and P. Öhberg, *Phys. Rev. A* **67**, 033602 (2003).
 - [6] R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati and I. Carusotto, *Phys. Rev. A* **78**, 021603 (2008).
 - [7] I. Carusotto, S. Fagnocchi, A. Recati, R. Balbinot and A. Fabri, *New J. Phys.* **10**, 103001 (2008).
 - [8] J. Macher and R. Parentani, *Phys. Rev. A* **80**, 043601 (2009).
 - [9] S. Finazzi and R. Parentani, *New J. Phys.* **12**, 095015 (2010).
 - [10] A. Coutant and R. Parentani, *Phys. Rev. D* **81**, 084042 (2010).
 - [11] O. Lahav, A. Itah, A. Blumkin, C. Gordon, S. Rinott, A. Zayats and J. Steinhauer, *Phys. Rev. Lett.* **105**, 240401 (2010).
 - [12] I. Zapata, M. Albert, R. Parentani and F. Sols, *New J. Phys.* **13**, 063048 (2011).
 - [13] P.-É. Larré, A. Recati, I. Carusotto and N. Pavloff, *Phys. Rev. A*, **85**, 013621 (2012).
 - [14] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, (Cambridge University Press, London, 1982).
 - [15] R. Loudon, *The Quantum Theory of Light*, 3rd ed., (Oxford University Press, New York, 2000).
 - [16] D. F. Walls and G. J. Milburn, *Quantum Optics*, 2nd ed., (Springer Verlag, Berlin, 2008).
 - [17] A. Recati, N. Pavloff and I. Carusotto, *Phys. Rev. A* **80**, 043603 (2009).
 - [18] R. Schützhold, *Phys. Rev. Lett.* **97**, 190405 (2006).
 - [19] E. Rubino *et al.*, *Phys. Rev. Lett.* **108**, 253901 (2012).
 - [20] D. Campo and R. Parentani, *Phys. Rev. D* **74**, 025001 (2006).
 - [21] E. Martín-Martínez, L. J. Garay and J. León, *Phys. Rev. D* **82**, 064028 (2010).
 - [22] N. Friis and I. Fuentes, *J. Mod. Opt.* (2012), in press (arXiv:1204.0617).
 - [23] Horstmann, B., Reznik, B., Fagnocchi, S. and Cirac, J.I., *Phys. Rev. Lett.* **104**, 250403 (2010).
 - [24] Horstmann, B., Schützhold, R., Reznik, B., Fagnocchi, S. and Cirac, J.I., *New J. Phys.* **13**, 045008 (2011).
 - [25] K. V. Kheruntsyan *et al.*, *Phys. Rev. Lett.* **108**, 260401 (2012).
 - [26] Note that, because of the condensate flow, in general $S_{ij} \neq S_{ji}$.
 - [27] I. Zapata and F. Sols, *Phys. Rev. Lett.* **102**, 180405 (2009).